Math 112 – Review for Test 2

Directions: Show your work for full credit. The way you drive your answer is most important.

1) Suppose y is a function of t whose derivative is given by

$$y' = 3y^2 \qquad \text{and that} \qquad y = -1 \text{ when } t = 0.$$

Write a loop program in Maple to approximate the value of y when

1. $t = 3$ (Use $\Delta t = 0.5$).
2. $t = -3$ (Use $\Delta t = -0.5$)
3. $t = 6$ (Use $\Delta t = 0.1$).

2) Write a loop program in Maple that performs the Euler's Method to approximate the value of y at $x = 1$ on the solution curve to the differential equation

$$\frac{dy}{dx} = x^3 - y^3$$

that passes through $(0, 0)$. Use $\Delta x = 0.2$.

3) Solve the following integrals with an appropriate technique.

1. $\int x^{99} \ln x dx$
2. $\int \frac{3x-2}{(x-1)(x^2+1)} dx$
3. $\int x\sqrt{2x+1} dx$
4. $\int x^2 \ln \sqrt{x} dx$
5. $\int_2^3 \frac{2x+3}{(x+1)^2} dx$
6. $\int_1^e \frac{\sin(\ln x)}{x} dx$
7. $\int \frac{e^{\tan x}}{1-\sin^2 x} dx$

4) Use integration by parts to show : $\int_a^\infty e^{-x^2} dx = \frac{e^{-a^2}}{2a} - \frac{1}{2}\int_a^\infty \frac{e^{-x^2}}{x^2} dx$. Hint: Choose $u = \frac{1}{x}$ and $dv = xe^{-x^2} dx$.

5) Find the arc length of the curve $f(x) = \frac{1}{3}(x-3)\sqrt{x}$ from $x = 0$ to $x = 3$.

6) Let $F(x) = \int_0^x \sqrt{e^{2t} - 1}dt$. Find the arc length of $F(x)$ for $0 \leq x \leq 2$.

7) The base of a certain solid is the circle $x^2 + y^2 = 36$ and each cross section perpendicular to the y-axis is an equilateral triangle. Find the volume of the solid.

8) Find the volume which results when the region bounded by $y = 1/x$ and $y = 1/x^2$ and the line $x = 2$ is revolved around the

1. x-axis.
2. y-axis.

9) Find the volume of the solid with the given information about its cross sections.

1. The base of the solid is an isoceles right triangle whose legs are each 5 units long. The cross sections parallel to one of the legs are semicircular.
2. The solid has a circular base with radius 2, and the cross sections perpendicular to a fixed diameter of the base are squares.
3. The base is an equilateral triangle with side length 10. The cross sections perpendicular to a given altitude of the triangle are semicircles.

10) Derive formulas for the volumes of the following solids.

1. A right circular cone with height h and radius (of the base) r.
2. The cap of a sphere resulting from slicing a sphere of radius r at a distance h from its center.
3. A right pyramid with square base of side length a and height h.

11) Find the area of the region bounded by the curves $y = 1 - 2x, \quad y = \sqrt{x}, \quad y = -x.$

12) Find the area of the following regions bounded by the given curves by two methods: a) integrating with respect to x, and b) integrating with respect to y.

i) $4x + y^2 = 0, \qquad y = 2x + 4$

ii)$x + 1 = 2(y - 2)^2, \qquad x + 6y = 7$

13) Find the length of the curve.

i) $y^2 = x^3$ between (1,1) and (4,8).

ii)$y = 6\sqrt{x}$, $1 \le x \le 4$.

iii)$12xy = 4y^4 + 3$ between $(\frac{17}{2}, 1)$ and $(\frac{67}{24}, 2)$

14) The base of a certain solid is the region bounded above by the line $y = 9$ and below by the graph of $y = 4x^2$. Cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

15) A solid has a circular base of radius 1. Parallel cross-sections perpendicular to the base are equilateral triangles. Find the volume of the solid.

16) For each of the following solids, set up integrals that give the volume using *both* the washer/disk method and the method of cylindrical shells. Then compute the integrals using Maple. Make sure both methods give the same answer.

i) The region bounded by $x = y^2$ and $x = -y$ rotated about x-axis.

ii) The region in i) rotated about the line $y = 5$.

iii) The region in i) rotated about $x = 5$.

iv) Solid obtained by rotating the region bounded by $y = \sqrt{x - 1}, y = 0, x = 5$ about the y-axis.

17) Find the area of the region bounded by the following curves both with respect to x and with respect to y.

1. $y = 2^x$, $y = 5^x$, $x = -1$, $x = 1$
2. $x = 9 - y^2$, $y = -3 - x$

18) Let R be a rectangle with vertices at the points $(a, 0)$, (a, h), $(b, 0)$, and (b, h), where $h > 0$ and $b > a$.

1. What is the area of R?
2. What is the volume of the solid obtained by rotating R about the x-axis?
3. What is the volume of the solid obtained by rotating R about the line $y = -c$, where $c \geq 0$?
4. What is the volume of the solid obtained by rotating R about the line $y = c$, where $c \geq h$?